The solar rotation rate from inversion of the first GONG datasets

T. Corbard[1], G. Berthomieu[1], J. Provost[1]

[1] Laboratoire G.-D.Cassini, Observatoire de la Cote d'Azur, Nice

ABSTRACT

This poster gives some results obtained by inverting the first GONG datasets for the rotational splittings with a full 2D Regularized Least-Squares (2D RLS) method. The resolution of the inversion is studied using averaging kernels and a low regularization leading to an upper limit of $0.1R_\odot$ for the width of the transition zone at the base of the convective zone. In addition, preliminary works on the introduction of a discontinuous basis of B-splines is presented and we show how this leads to averaging kernels that are better localized on each side of the discontinuous breakpoint.

1. Basic equations and inverse method

For a mode of oscillation (n,l), the observed frequency shift due to the angular velocity $\Omega(r,\mu)$ (r being the normalized solar radius and μ the cosine of the colatitude) is related to the solar model by the linear expression:

$$\Delta\nu_{nlm} = \int_{-1}^{1}\int_{0}^{1} K_{nlm}(r,\mu)\Omega(r,\mu)drd\mu \qquad (1)$$

where the kernel $K_{nlm}(r,\mu)$ associated with the mode depends on the model and is assumed to be known exactly.

The inverse method used is a 2D RLS method in which we search the unknown rotation as a linear combination of piecewise polynomials, projected on a tensorial product of B-spline basis (Corbard et al., 1996). The distribution of break points in radius is calculated such that the number of observed modes having their turning points between two consecutive break points is constant. We can define at each break point the kind of connection which is required between the right and the left pieces of polynomials. Formally, the rules of connection can differ from one break point to the next: at some of them, the polynomials can be discontinuous, at some others the left and right pieces can be tied to fulfilling the continuity of their first derivatives or only of the polynomials themselves, ...etc. In this work this possibility is used to allow the solution to be discontinuous at a given break point. In this case the regularization is applied on the left and right sides of the break point separately.

The spatial resolution of the inversion at a point (r_0, μ_0) can be estimated by the the radial and latitudinal full widths at mid height of the so called averaging kernels $\kappa(r_0, \mu_0, r, \mu)$ defined in Corbard et al. (1996).

2. GONG datasets used

The first dataset (dataset I hereafter) is a combination of both concatened spectra of GONG months 4 to 8 for degrees lower than $l = 30$ and averaged spectra concerning GONG months 4 to 7 for degrees up to $l = 150$. It contains 1987 modes with $1200 < \nu_{nl} < 5000$ μHz. Splittings are given by their expansion on Legendre polynomials with coefficients up to a_9.

The second dataset (dataset II) has been obtained from averaged spectra of GONG months 4 to 10 for degrees $2 \le l \le 150$. It contains 2103 modes (n,l) $l \le 150$ with $1200 < \nu_{nl} < 5000$ μHz. Splittings are given by their expansion on orthogonal polynomials with coefficients up to a_9. The orthogonal polynomials are defined in Ritzwoller & Lavely (1991).

In both cases individual splittings and their corresponding errors have been calculated from their expansion on polynomials and inversions have been performed from these individual splittings.

3. Results from dataset I and low regularization

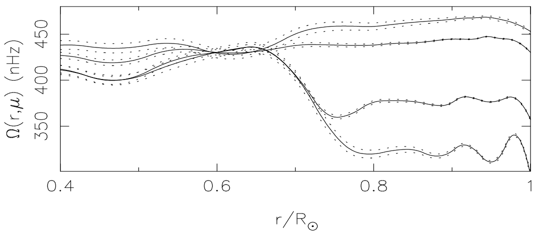

Figure 1: Rotation rate versus solar radius for latitudes $0, 30, 60, 90°$ obtained with dataset I

Figure 1 shows the inferred rotation rate for a choice of trade-off parameters that tends to minimize the χ^2 value rather than the regularization term. This leads to a solution with some oscillations especially in zones near the solar pole where the solution is not well constrained by the data but the transition zone is less smoothed by the regularization term. Compared to results obtained with an optimal choice of trade-off parameters based on an L-curve analysis (Corbard et al. 1996), this inferred rotation rate has a sharper transition zone (about $0.10R_\odot$) and the averaging kernels at this location ($r = 0.7R_\odot$) are better peaked with an increasing radial resolution ($\Delta r = 0.08R_\odot$ at mid latitude). Inversion performed with lower regularization doesn't enable to produce better radial resolution and increases the effect of input errors. Thus, this inversion is the one that gives the best radial resolution at this target location without increasing too much the effect of input errors. The fact that the width of the transition zone decreases when the resolution increases allows us to say that the transition occurs in a zone between 0.66 and 0.76 solar radius but it might be sharper.

At $0.55R_\odot$ averaging kernels are, for this inversion, sufficiently well localized in latitude so that the latitudinal independence of the inferred rotation rate at this depth can be considered as real. Below this depth we find again a latitudinal dependence but this may be an artifact of the errors and this latitudinal dependence below $0.55R_\odot$ is of lower importance with dataset II.

4. Preliminary works using dataset II and a discontinuous Basis of B-splines

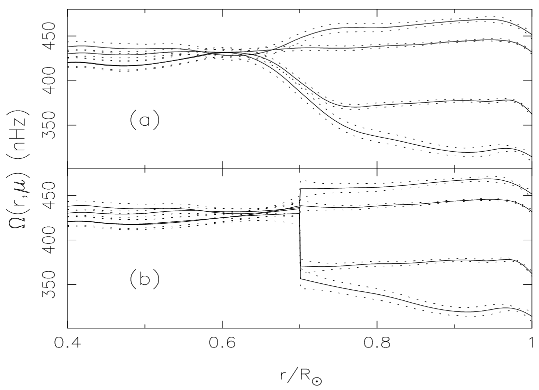

Figure 2: Rotation rate versus solar radius for latitudes $0, 30, 60, 90°$ obtained with dataset I. (a) Without discontinuous basis (b) With a basis discontinuous at $0.7R_\odot$

The solution obtained from dataset II is shown on the Fig. 2a for radius above $0.4R_\odot$. The inferred rotation rate is in good agreement with the previous one for latitudes below 60° and is less oscillating near the pole. As the observed modes and the trade-off parameters are the same as for dataset I, we can think that these zones are now better constrained by the data. Nevertheless we can not exclude that the regularization acts not exactly by the same way on the two datasets so that the regularization could be more important with dataset II.

We are studying the possibility of using a discontinuous basis of B-splines to estimate the thickness of the transition zone. An example is given on the Fig. 2b that shows the solution obtained when we allow the rotation to be discontinuous at the base of the convection zone ($0.7R_\odot$). The χ^2 value obtained for this solution and the resolutions are almost the same than with the standard basis indicating that the data are compatible with a very sharp transition zone.

Averaging kernels obtained at the equator and $0.69R_\odot$ are shown on Fig. 3 (full lines) for the solutions obtained with and without setting a discontinuous basis at $0.7R_\odot$. Although, without discontinuous basis the averaging kernel was equally distributed around $0.7R_\odot$, with a discontinuous basis the major part of the kernel is below $0.7R_\odot$. By this way the value of the rotation rate obtained at $0.69R_\odot$ is less influenced by the true rotation above $0.7R_\odot$ and this could explain its lower value in Fig. 2b than in Fig 2a. This may argue in favor of a solid rotation up to $0.69R_\odot$ but this conclusion would have been more reliable if the averaging kernel had strictly no part above $0.70R_\odot$. Furthermore the new averaging kernel is more oscillating between 0.72 and $0.9R_\odot$ so that its interpretation is harder. A symmetrical example is shown on Fig. 3 (dashed lines) by the averaging kernels calculated at the equator and $0.71R_\odot$ with and without allowing the rotation to be discontinuous at $0.7R_\odot$. This could explain by similar way the higher value obtained just above $0.7R_\odot$ when we perform the inversion with a discontinuous basis.

This work is very preliminary and we think that numerical experiments on the reconstruction of some arbitrary given test rotation rates should be useful to calibrate the effect of introducing a discontinuous basis.

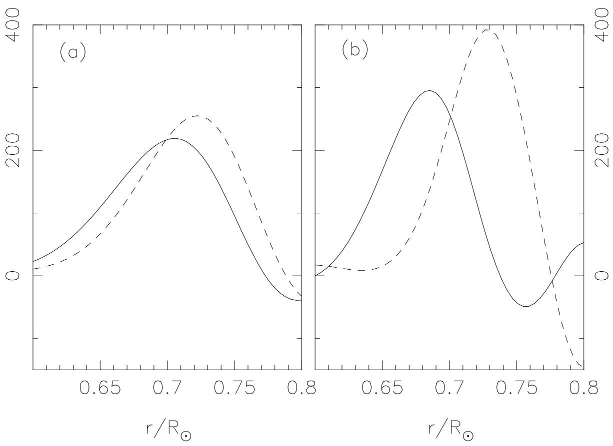

Figure 3: Averaging kernels computed at the equator and $0.69R_\odot$ (full lines) or $0.71R_\odot$ (dashed lines). (a) Without discontinuous basis, (b) With a basis discontinuous at $0.7R_\odot$

REFERENCES

1. Corbard T., Berthomieu G., Provost J., et al., 1996, submitted to A&A

2. Ritzwoller M.H., Lavely E.M., 1991, ApJ 369, 557