ABOUT STRUCTURE INVERSIONS OF SIMULATED COROT DATA FOR A SOLAR LIKE STAR

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ABSTRACT

We study the potential of exploring the internal structure of solar like stars which could be observed by COROT. We consider a solar like star of mass and age which allows stochastically excited oscillations according to the results of Houdek *et al* (1999). Taking into account COROT observational constraints, we estimate a set of observable modes with the uncertainties on their frequencies; then, in the same way followed by Gough and Kosovichev (1993), we evaluate the possibility of inferring structure properties of the star using inversion techniques.

Key words: Stars: oscillations.

1. PROPERTIES OF THE CONSIDERED SOLAR LIKE STAR

The future asteroseismic space missions like COROT, MONS, MOST, EDDINGTON,... (see session IV of these proceedings) will give access to an increasing number of stellar oscillation frequencies with good accuracy. To prepare the interpretation of these data in terms of internal structure of the stars, we present the results of the inversion of frequencies of simulated "COROT" data from a solar-like star. We consider a solar-like star with a 1.45 solar mass, an effective temperature $T_{eff} = 6567$ K, and an age of 1.63 10^9 years. It has a convective core with radius 0.085 $R_\star$ and a central hydrogen abundance of $X_c = 0.3$.

2. SIMULATED PERIODOGRAM

In a first step, we try to predict the set of modes which will be detectable and their errors. A simulated spectrum is constructed assuming a total observation time of $T_{tot} = 150$ days which corresponds to a spectrum resolution of $\Delta\nu_0 = 1/T_{tot} = 0.077\ \mu$Hz.

The signal is composed of a set of stochastically excited and intrinsically damped harmonic oscillators corresponding to the spatially filtered global p modes of degrees $l = 0, 1, 2, 3$.

The analytic expression of the profile of each mode in the power spectrum $P(\nu)$ is given in table 1. The quantity a is related to the averaged power spectrum of the forcing function and $1/\eta$ is the life time of the mode of frequency ν_0. The height of the maximum $P(\nu_0)$ is related to the luminosity fluctuation $\delta L/L$ of the corresponding mode. We use the predictions of Houdek *et al* (1999) on the amplitudes and life time of the modes excited by convection.

Table 1.

	Profile of a mode
$\eta \gg \Delta\nu_0$	$P(\nu) = \dfrac{a^2}{4\eta^2\left[1+\left(\frac{\nu-\nu_0}{\frac{\eta}{2\pi}}\right)^2\right]}$
$\eta \ll \Delta\nu_0$	$P(\nu) = \dfrac{a^2T_{tot}^2}{4}\dfrac{\sin^2[T_{tot}\pi(\nu-\nu_0)]}{[T_{tot}\pi(\nu-\nu_0)]^2}$
	Height of the maximum
$\eta \gg \Delta\nu_0$	$P(\nu_0) = \left(\dfrac{\delta L}{L}\right)^2\dfrac{2}{\eta\Delta\nu_0}$
$\eta \ll \Delta\nu_0$	$P(\nu_0) = \left(\dfrac{\delta L}{L}\right)^2\dfrac{1}{\Delta\nu_0^2}$

We take into account a COROT noise level $B = 0.6$ ppm for 5 days observation corresponding to a noise level in the periodogram of $b \sim 0.16\,\mathrm{ppm}^2/\mu\mathrm{Hz}$. The stellar background power spectrum is derived from Trampedach *et al.* (1998)

3. SELECTION OF MODES EXPECTED TO BE DETECTED

The selection of the modes expected to be detected is made as follows. Let $P_{b\alpha}$ be the level corresponding to the probability α that a peak higher than $P_{b\alpha}$ in a given bandwidth (with $N-1$ bins) in the periodogram is due to noise:

$$P_{b\alpha} = b \ln \left[\frac{N-1}{\alpha}\right].$$

The probability that a mode of amplitude $A_{n,l,m}$ has a maximum height $P_{n,l,m}$ in the periodogram larger than $P_{b\alpha}$ is given by:

$$\mathcal{P}_{A_{n,l,m}} = \mathcal{P}(P_{n,l,m} > P_{b\alpha}) = e^{-\frac{P_{b\alpha}}{b+A_{n,l,m}}}.$$

We take $\alpha = 10\%$ and a bandwidth of 35 μHz corresponding to about half the mean large separation between the p-mode frequencies of same degree and consecutive radial orders.

The level $P_{b\alpha}$ is plotted in figure 1. The frequencies of the selected modes, plotted in figure 2 a,b relatively to their turning points, correspond to $\mathcal{P}_{A_{n,l,m}}$ larger than 90% and 50% respectively.

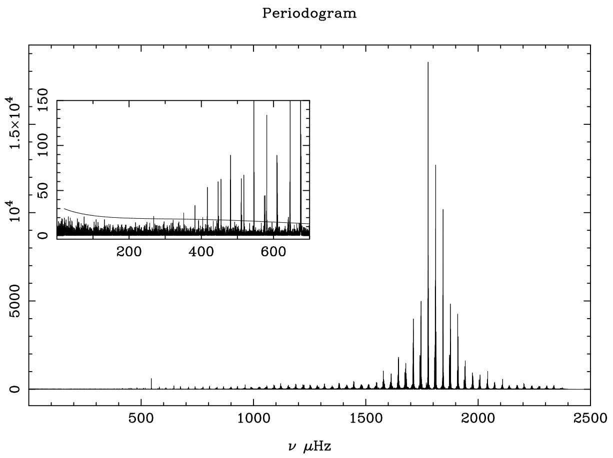

Figure 1. Simulated periodogram (in ppm$^2/\mu$Hz). In the low frequency range, the level $P_{b\alpha}$ is indicated ($\alpha = 10\%$).

The expected error on the central frequency ν_0 is derived by the method of Toutain and Appourchaux (1994) which uses the maximum likelihood estimators to fit the peaks in the spectrum to Lorentzian profiles. For the simplest case of a unique mode with a Lorentzian profile it reduces to the expression of Libbrecht (1992):

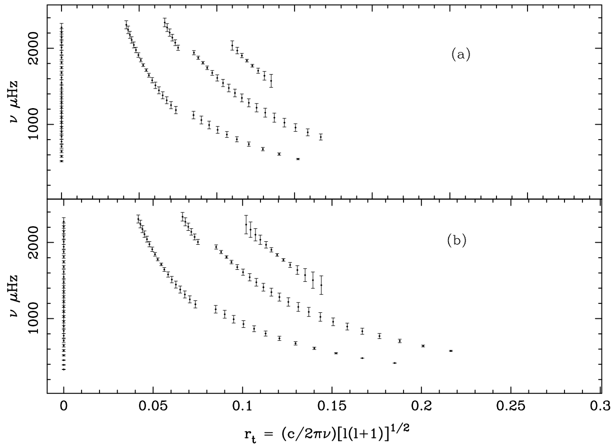

Figure 2. Frequencies in μHz of selected modes versus their turning point: (a) set of modes with $P_{a_{n,l,m}} > 90\%$; (b) set of modes with $P_{A_{n,l,m}} > 50\%$. The expected errors on the frequencies (multiplied by 200) are indicated. They are minimum around the maximum of the signal ($\nu_0 \simeq 1800$ μHz) and they decrease at low frequency when the mode lifetime increases.

$$\sigma_{\nu_0} = \sqrt{\frac{\Gamma}{4\pi T}\sqrt{\beta+1}(\sqrt{\beta+1}+\sqrt{\beta})^3},$$

where Γ is the line-width at half maximum and β the noise-to-signal ratio. They are indicated in figure 2.

4. PRELIMINARY RESULTS ON AVERAGING KERNELS OF SOLA INVERSION OF THE SELECTED MODES

There are many ways to extract information about the internal structure of the star from its frequencies. For example, the "large" separation is a global measure of the sound travel time through the star, the "small" separation informs about the core structure (see Gough 1998). Here we focus on the potential of inversion methods used in helioseismology to infer internal sound speed of the star.

The linearization of the equations of stellar oscillations around a given reference model leads to:

$$\frac{\delta\omega_i}{\omega_i} = \int_0^1 K^i_{c,\rho}\frac{\delta c}{c}\mathrm{d}x + \int_0^1 K^i_{\rho,c}\frac{\delta\rho}{\rho}\mathrm{d}x + \frac{F(\omega_i)}{Q_i} - \frac{3}{2}\frac{\delta R_\star}{R_\star} + \frac{1}{2}\frac{\delta M_\star}{M_\star} + \varepsilon_i, \tag{1}$$

where $x = \dfrac{r}{R_\star}$; $\dfrac{\delta c}{c}$, $\dfrac{\delta\rho}{\rho}$, $\dfrac{\delta R_\star}{R_\star}$ and $\dfrac{\delta M_\star}{M_\star}$ are the relative differences of sound speed, density, radius and mass between the real star and its model; $K^i_{c,\rho}$ and

$K^i_{\rho,c}$ are the respective kernels for sound speed and density; $F(\omega_i)$ is a term taking into account the effect on frequencies of the surface uncertainties; Q_i is the energy of the mode divided by the energy that a radial mode ($l = 0$) at the same frequency would have; ε_i is the error in the data assumed to be independent and Gaussian distributed with variances σ_i^2.

The aim of SOLA inversion, at given target point $x_0 = r_0/R_\star$, is to find a set of inversion coefficients $\alpha_i(x_0)$ such that the mean value of the sound speed difference can be expressed as a linear combination of the data:

$$\sum_{i=1}^{N} \alpha_i(x_0) \frac{\delta\omega_i}{\omega_i} =< \frac{\delta c}{c}(x_0) > .$$

The kernel involved in the averaging of $\frac{\delta c}{c}(x_0)$ is given by

$$\overline{\mathbf{K}}(r, x_0) = \sum_{i=1}^{N} \alpha_i(x_0) K^i_{c,\rho}(r).$$

The variance of the value obtained for $< \frac{\delta c}{c}(x_0) >$ is given by:

$$\sigma(x_0)^2 = \sum_{i=1}^{N} \alpha_i(x_0) {\sigma_i}^2.$$

The averaging kernel $\overline{\mathbf{K}}(r, x_0)$ is chosen by least square minimization as close as possible to a well localized target kernel:

$$T(x_0, x) = xD \exp\left(-\frac{x - x_0}{\Delta}\right)^2 .$$

Additionnal constraints are included to suppress the surface effects, to minimize the contribution of $\frac{\delta\rho}{\rho}$ (cross-term) and to minimize the error propagation.

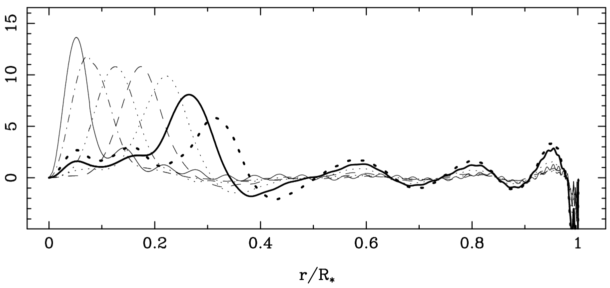

Figure 3. Averaging kernels for $x_0 = 0.025, 0.075, 0.125, 0.175, 0.225, 0.275, 0.325$.

The different parameters of the inversion needed to construct the averaging kernels are selected in the same way as in Rabello-Soares *et al.* (1999). The estimation of the quality of the inversion is given by the measure of the "distance" between the averaging kernel and the target:

$$\chi = \int_0^1 \left[\overline{\mathbf{K}}(x, x_0) - T(x_0, x)\right]^2 \mathrm{d}x.$$

An estimation of the contamination of the results on $< \frac{\delta c}{c}(x_0) >$ by the contribution of $\frac{\delta\rho}{\rho}$ to the frequency differences is given by the "cross-term"

$$C(x_0) = \sqrt{\int_0^1 \left[\sum_{i=1}^{N} \alpha_i K^i_{\rho,c}(x)\right]^2 \mathrm{d}x}.$$

Preliminary results on the averaging kernels using the selected modes with $P_{A_{n,l,m}} > 50\%$ and their errors are given in figures 3 and 4. Their widths are defined by the distance between the quartile points Δq.

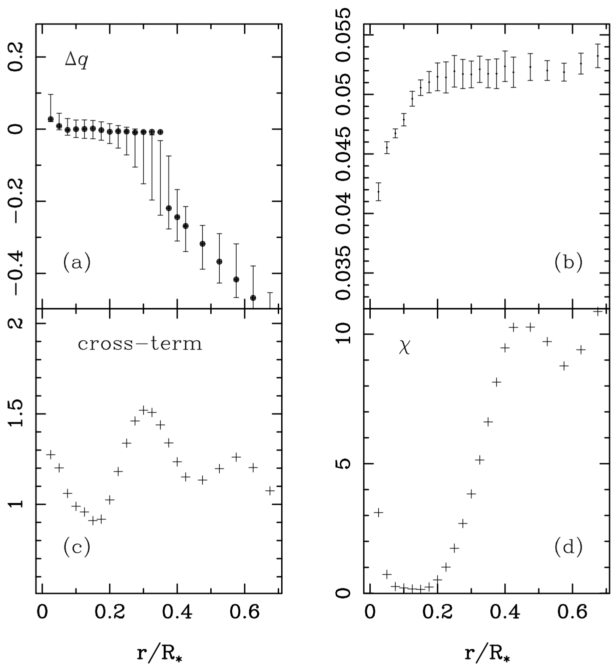

Figure 4. Variation with the position x_0 *of: (a) the distance between the location of the maximum of the kernel and* x_0 *(the width of the kernel* Δq *is indicated by the vertical bar); (b) the width of the target kernel* Δ *and the variance of the error* $\sigma(x_0)$ *indicated by the vertical bar; (c) the cross term; (d) the quantity* χ.

It appears that fairly well localized averaging kernels are available only in a small domain in the core of the star. From figure 4 (a), it is seen that the averaging kernels are well localized (small Δq) and the location of their maximum is close to the target x_0 only for $0.05 < x < 0.4$. Moreover, figure 4 (d) shows that the averaging kernels are close to the target one (small χ) only for $x_0 < 0.3$ and figure 4 (c) shows that this domain corresponds to the minimum of the cross term. For larger values of x_0, the kernels are spread out with an increase of their width Δq.

5. ACKNOWLEDGEMENTS

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