Analysing the Meridional Circulation using Fourier-Hankel-Decomposition

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Content

Outline

Reduction of observed data with the FHD

Partition of the residual Doppler-Signal Ψ

$$\Psi(R,\theta,\varphi,t) = \sum_{L,m,\nu} e^{\imath(m\varphi+\nu t)} \left[A_{L,m,\nu}\cdot\Theta_L^m(\cos\theta) + \ldots\right.$$

$$\ldots \qquad \left. + B_{L,m,\nu}\cdot\overline{\Theta_L^m(\cos\theta)}\right]$$

$$\Theta_L^m(\cos\theta) := N_L^m \cdot \left[P_l^m(\cos\theta) - \frac{2\imath}{\pi} Q_l^m(\cos\theta)\right]$$

Spectra

Frequency-spectra $A_{L,m,\nu}$ and $B_{L,m,\nu}$ by integration over θ, φ, t

Shift between the spectra $A_{L,m,\nu}$ and $B_{L,m,\nu}$

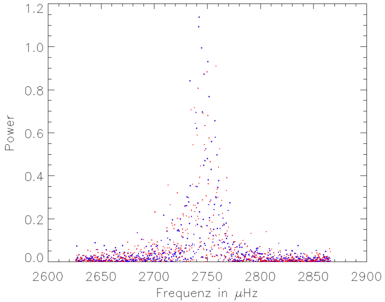


Figure: Example for a peak in the spectra of pole- and equatorward flow.

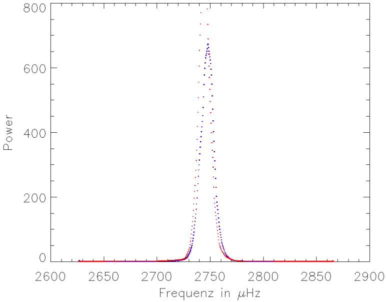


Figure: Example for smoothed peak, showing a frequency-shift between both spectra.

Estimation of the velocity-profile

Assumption

The value $(U/r)_{n,L}$ is constant over the range spanned by the penetration-depth of the mode (n,L) !

Estimation

$$\Delta\nu_{n,L} = \frac{I \int\limits_0^{R_\odot} \rho_0 \left(\frac{U}{r}\right)_{n,L} \cdot K_{n,L,m}(r)\,\mathrm{dr}}{\pi \int\limits_0^{R_\odot} \rho_0 \cdot K_{n,L}(r)\,\mathrm{dr}} = \frac{I}{\pi}\,\frac{U(R_\odot)}{R_\odot}$$

$$U(R_\odot) = \pi R_\odot / I \cdot \Delta\nu_{n,L} =: U'$$

Estimation of the velocity-profile

Estimation

- Obtain set of points $\{(\nu_i/L_i, U_i')\}$
- Find relation: „penetration-depth“ $\epsilon \leftrightarrow \nu/L$
- Resulting in velocity profile $U'(\epsilon)$

Outline

Data

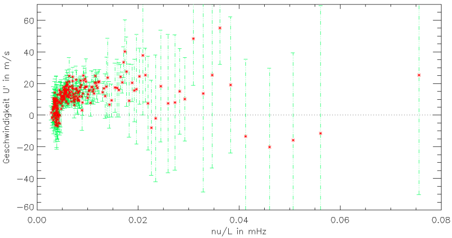
- Dopplergrams of SOI/MDI (SOHO)
- Full-disk-images – from 01.04.99 to 30.04.99
- Transformed to equidistant θ-φ-lattice

Parameters for integration

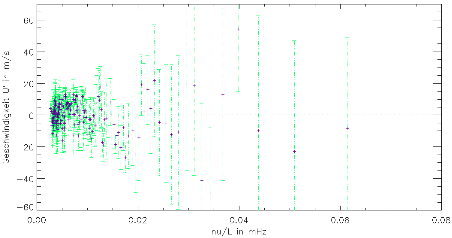
- m=0
- $L = 13 + 2 \cdot j \quad , \quad j \in \{0, \ldots, 504\}$
- Resolution in ν: 0.4 μHz
- Interval of the polar angle $\Theta = \pi/4$

Outline

Velocity profile over ν/L

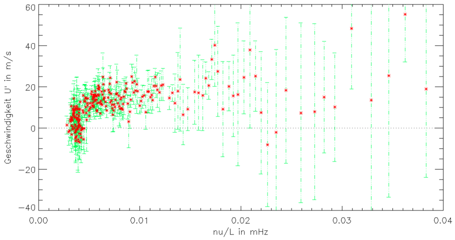


Northern hemisphere, binned velocity profile over ν/L

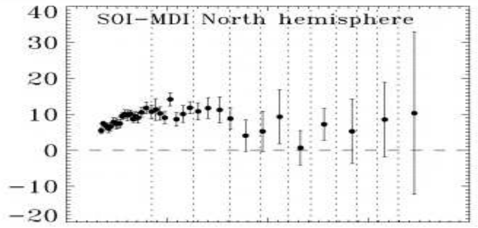


Southern hemisphere, binned velocity profile over ν/L

Velocity profile over ν/L: comparison with known results

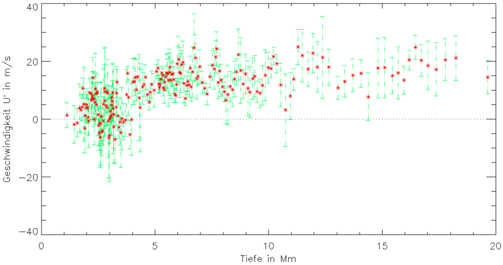


Northern hemisphere, velocity over ν/L, comparable part



Results of D. Braun 1999, velocity over ν/L, northern hemisphere

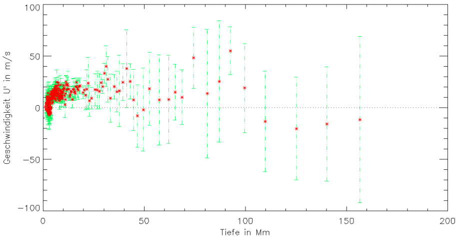
Velocity profile over depth: comparison with known results



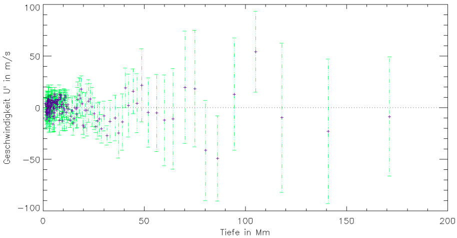
Northern hemisphere, velocity over penetration depth ϵ, comparable part with ϵ from 0 to 20 Mm, average velocity of (15 ± 5) m/s

According to eg.(Zhao, Kosovichev, Duvall – 2004) the velocity of the poleward-flow in the region of $\theta \approx 45^{\circ}$ is $(10 - 15)$ m/s. This velocity is constant in the range of depth of around 14 Mm.

Velocity profile over depth - extended to 170 Mm

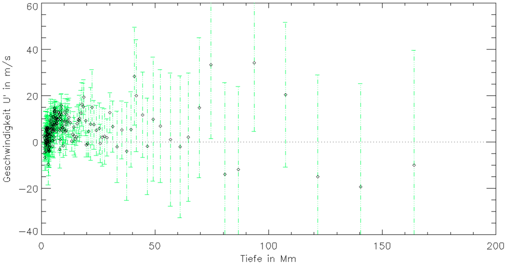


Northern hemisphere, velocity over ν/L



Southern hemisphere, velocity over ν/L

Profile over depth - average profile



Mean velocity profile of both hemispheres down to $\epsilon =$170 Mm

Outline

Conclusions

What did we get?

- Rough estimation for the quality of the profile
- Agreement with known results in shallow layers
- Agreement with known results by same method in middle-deep layers
- Indications for return flow in layers of around 130 Mm depth

Ideas

How to improve?

- Longer time-series for better resolution
- Integration over bigger set of data (esp. $m \neq 0$)
- Variation of θ
- Time-varying survey
- Better detection of peaks in the frequency-spectra, increasing the number of $\Delta\nu_i$

Dopplergrams – original and transformed

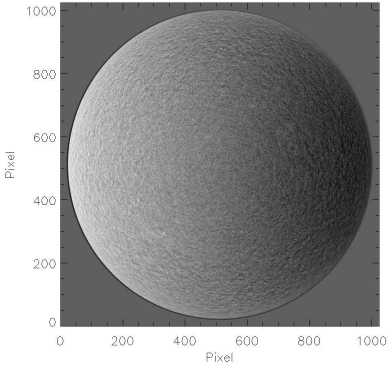


Figure: Example for an original dopplergram.

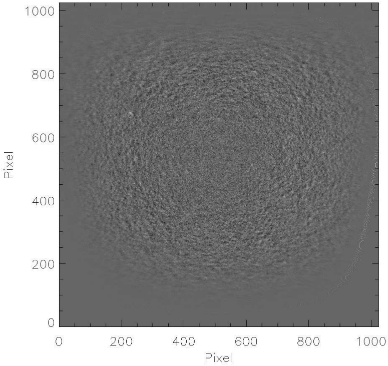
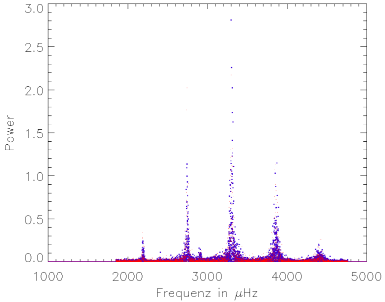
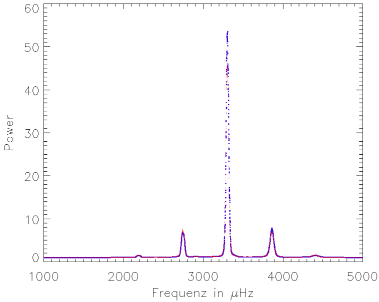


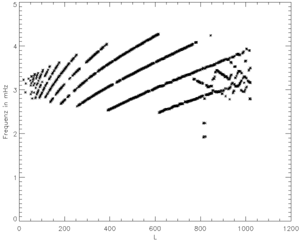
Figure: Reduced example-dopplergram on θ-φ-lattice.

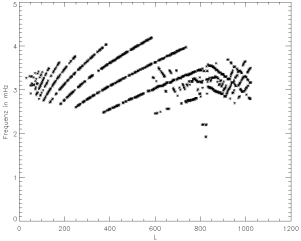
Spektra – total





L-ν-diagrams





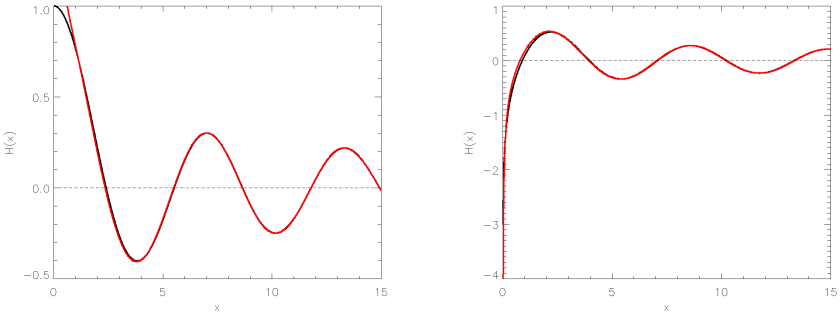
Hankel-approximation

Hankel

- Idea: take Hankel $H_m^{(1,2)}(L\theta)$ as solution of radial oscillation ODE
- Approximation for $l \gg 1$ and $l \gg m$ (far-field-approx.)

$$H_m^{(1,2)}(L\theta) \approx (-1)^m \frac{(l-m)!}{(l+m)!} \left[P_l^m(\cos\theta) \pm \frac{2\imath}{\pi} Q_l^m(\cos\theta) \right]$$

Hankel1



Hankel2